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HIGH ENERGY CHARGED PARTICLE OPTICS COMPUTER PROGRAMS

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The computer programs TRANSPORT and TURTLE are described, with special emphasis on recent developments. TRANSPORT is a general matrix evaluation and fitting program. First and second-order transfer matrix elements, including those contributing to time-of-flight differences can be evaluated. Matrix elements of both orders can be fit, separately or simultaneously. Floor coordinates of the beam line may be calculated and included in any fits. Tables of results of misalignments, including effects of bilinear terms can be produced. Fringe fields and pole face rotation angles of bending magnets may be included and also adjusted automatically during the fitting process to produce rectangular magnets. A great variety of output options is available.

TURTLE is a Monte Carlo program used to simulate beam line performance. It includes second-order terms and aperture constraints. Replacable subroutines allow an unlimited variety of input beam distributions, scattering algorithms, variables which can be histogrammed, and aperture shapes. Histograms of beam loss can also be produced. Rectangular zero-gradient bending magnets with proper circular trajectories, sagitta offsets, pole face rotation angles, and aperture constraints can be included. The effect of multipple components of quadrupoles up to 40 pole can be evaluated.

Introduction

A complete design of a beam line for transmission of charged particles involves two stages. First, one must determine certain quantifiable characteristics that the beam line must possess, and produce a design which optimizes the conformance to these characteristics. Second, one must evaluate the performance of the system produced. The latter might involve a determination of beam profiles, acceptances, and effects of magnet imperfections.

We describe here two computer programs developed to achieve the two purposes described above. A beam design, including all element spacings and magnetic fields, in produced using the program TRANSPORT.[1] Once this design is achieved, it may be simulated using the Monte Carlo program TURTLE.[2] The two programs use the same input data format making the transition from one to another quite simple.

Both programs are described in detail in their respective manuals[1,3], and, to some extent, in the published literature. Their use is sufficiently widespread so there is no point in giving a detailed description of either. For completeness, we give a short description of each, with a greater elaboration of recent developments. Some of the more recent developments have been or will be published elsewhere. Others will appear only in this article.

TRANSPORT

TRANSPORT is a general matrix evaluation and fitting program. It can evaluate various matrices which represent the transmission of particles through a beam line, and vary the physical parameters of the beam line to fit elements of such matrices to desired values. A schematic illustration of a beam line is shown in figure 1.

TRANSPORT considers a beam line to be comprised of a set of magnetic elements placed sequentially at intervals along an assumed reference trajectory. The reference trajectory is taken to be a path of a charged particle passing through idealized magnets (no fringing fields) and having the central design momentum of the beam line. Therefore, through a bending magnet, the reference trajectory is the arc of a circle, while through all other magnetic elements it is a straight line. The input data to TRANSPORT contain the initial floor coordinates and direction of the reference trajectory, and the sequence of elements comprising the beam line. The elements include both drift spaces and magnetic elements, which are specified by their lengths, magnetic fields, and other relevant quantities. TRANSPORT can then calculate the floor coordinates of the reference trajectory at the interface between any two elements.

A local coordinate system is attached to each point on the reference trajectory. As a particle moves down the beam line, its transverse position and direction of motion are referred to this local coordinate system. An illustration of this local coordinate system is shown in figure 2. A six component vector is used to describe a particle trajectory at a given position along the beam line, i.e.

$$X = \begin{bmatrix} x \\ \theta \\ y \\ \phi \\ \ell \\ \delta \end{bmatrix} \tag{1}$$

where:

x = the horizontal displacement of the ray with respect to the reference trajectory

θ = the angle the ray makes is the horizontal plane with the reference trajectory

y = the vertical displacement of the ray with respect to the reference trajectory

ϕ = the verticle angle the ray makes with the reference trajectory

ℓ = the longitudinal separation between the ray and the central trajectory

δ = $\Delta p/p$ is the fractional momentum deviation of the ray from that of the reference trajectory.

The value of this vector at any location is the beam line may be determined from its initial value by means of a transfer matrix R, so that

$$X(1) = R\ X(0) \tag{2}$$

where the arguments (0) and (1) indicate the initial location and the point of interest, respectively. The six by six matrix R takes on a simple form if the system has midplane symmetry, where all the magnetic potentials are odd in the vertical coordinate. Then we have

$$\begin{pmatrix} x_1 \\ \theta_1 \\ y_1 \\ \phi_1 \\ \ell_1 \\ \delta \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ \theta_0 \\ y_0 \\ \phi_0 \\ \ell_0 \\ \delta \end{pmatrix} \tag{3}$$

In all cases, whether or not we have midplane symmetry, all entries in the fifth column, except the fifth, and all entries in the sixth row, except the sixth, will be zero. Thus nothing affects the momentum, and the longitudinal separation affects no transverse coordinate. TRANSPORT can print the transfer matrix at any or all locations in matrix format or in a single line which contains only those elements in the first four rows which survive midplane symmetry.

The matrix formalism can be regarded as the first term in a Taylor's series and extended to second order via the equation

$$x_i(1) = \sum_j R_{ij} x_j(0) + \sum_{jk} T_{ijk} x_j(0) x_k(0) \qquad (4)$$

The second-order matrix elements which contribute to the transverse coordinates have been calculated by Brown,[4] and have previously been published. The terms which contribute to longitudinal separation have been derived by several interested parties, and have been[5] or will be published.[6]

In accelerator and beam transport systems, the behavior of an individual particle is often of less concern than is the behavior of a bundle of particles (the beam), of which the individual particle is a member. An extension of the matrix algebra of eq. (2) provides a convenient means for defining and manipulating this beam. TRANSPORT assumes that the beam may be correctly represented in phase space by an ellipsoid in the six-dimensional coordinate system described above. Particles in a beam are assumed to occupy the volume enclosed by the ellipsoid, each point representing a possible ray. The sum total of all phase points, the phase space volume, is commonly referred to as the "phase space" occupied by the beam. A diagram of a two-dimensional cross section of this six-dimensional ellipsoid is shown in figure 3.

The equation of the six-dimensional ellipsoid is

$$X^T\sigma^{-1}X = 1 \tag{5}$$

where σ is the beam or sigma matrix, some of whose elements are found in the illustration. The correlation terms r are given in terms of the off-diagonal elements of σ by

$$r_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}} \tag{6}$$

The ellipse at one location in the beam line can be transformed into one at another location by means of the transfer matrix between the two locations, so that

$$\sigma_{ij} = \sum_{k} R_{ik}R_{j\ell}\sigma_{k\ell}(0) \tag{7}$$

An alternate interpretation of the sigma matrix is that is provides the second moments of a six-dimensional Gaussian distribution. The fourth moments can then be calculated and second order terms taken into account. The sigma matrix now transforms as

$$\overline{X}_i(1) = \sum_{jk} T_{ijk}\sigma_{jk}(0) \tag{8}$$

$$\sigma_{ij}(1) = \sum_{k} R_{ik}R_{j\ell}\sigma_{k\ell}(0) + 2\sum_{\ell m}\left(\sum_{k} T_{ik\ell}\sigma_{km}(0)\right)\left(\sum_{n} T_{jmn}\sigma_{\ell n}(0)\right)$$

The distribution is no longer an ellipsoid and the centroid at the final position is no longer the image of the centroid at the initial position. Nevertheless, the sigma matrix does provide an estimate of the beam dimensions and is of use in determining the magnitude of the net contribution of second-order aberrations. The second order transformation of the sigma matrix has been described elsewhere.[7]

A number of the physical parameters describing the magnets or their locations or orientations may be varied by the program. A list of elements which are physical or have parameters which may be varied is given in table 1. Almost all items require no explanation. The single possible exception is a bending magnet, whose configuration can sometimes be quite complicated.

A bend magnet element specifies a sector bend magnet where the field boundaries are infinitely sharp and form a plane perpendicular to the reference trajectory at the input and output faces of the magnet. A field boundary making an angle with the perpendicular plane is specified as a separate element which precedes or follows the magnet. The rotation of the field boundary acts as a quadrupole component which can affect the first-order transfer matrix. A quadratic variation of the central field of a bend magnet, or a curvature of its entrance or exit field boundaries can contribute a sextupole component which will affect the second-order transfer matrix. A diagram of a general bending magnet is shown in figure 4.

The values of the parameters to be varied will be found which will satisfy any user-imposed constraints. A variety of constraints is available and a list is given in table II. Any assortment of constraints can be fit simultaneously by TRANSPORT, providing the configuration is physically possible. First- and second-order constraints may be mixed and all parameters describing magnetic elements or their intended location or orientation may be varied in either a first- or second-order run. The only exceptions are that parameters which affect only the second-order characteristics of a beam may not be varied and second-order constraints may not be imposed in a first-order run. Parameters directly describing the beam ellipse and misalignment parameters may be varied only in first order.

The misalignment tolerances of the magnets in a beam line can also be determined by TRANSPORT. The complete theory of magnet alignment tolerances has been given elsewhere[8] and will be described briefly here. A picture of a misaligned bending magnet is shown in figure 5.

The most immediate effect of the misalignment of a magnetic element is a displacement of the reference trajectory. This would add a term to equation (2) which was not dependent on the values of the initial coordinates of the trajectory.

However, many possible misalignments, such as the rotation of a quadrupole about its axis would not be included in such a term. We therefore add a second term which is bilinear in the extent of the misalignment and arrive at

$$X(1) = RX(0) + Fm + GX(0)m \tag{9}$$

where m is a vector of misalignment parameters given by

$$m = \begin{bmatrix} \delta x \\ \theta_x \\ \delta y \\ \theta_y \\ \delta z \\ \theta_z \end{bmatrix} \tag{10}$$

The six components indicate displacements and rotations with respect to the three axes of the reference coordinate system at the entrance face of the magnet.

In TRANSPORT a number of possible elements or portions of the beam line may be misaligned. An individual element or section of a beam line can be misaligned, and misalignments can be nexted. Also TRANSPORT can be instructed to misalign all quadrupoles and/or bending magnets by a given amount.

The effects of misalignments are shown in the beam matrix. The misalignments may be of two types. A known misalignment of a magnet will produce a displacement of the beam centroid. The new beam centroid and sigma matrix are given by

$$\bar{X} = Fm$$

$$\sigma(1) = R\sigma(0)R^T + G\sigma(0)mR^T + R\sigma(0)m^TG^T + G\sigma(0)mm^TG^T \qquad (11)$$

An uncertainty in position will not affect the beam centroid, but will produce an altered beam ellipse given by

$$\sigma(1) = R\sigma(0)R^T + F\langle mm^T\rangle F^T + G\sigma(0)\langle mm^T\rangle G^T \qquad (12)$$

The matrix $\langle mm^T\rangle$ represents an ellipsoid of uncertainty in the six-dimensional space of misalignment parameters. If the misalignments are uncorrelated this ellipsoid will be upright. If the initial dimensions of the beam ellipse are zero, then the beam matrix will represent the envelope of possible locations of the reference trajectory.

The results of the misalignments may be represented in either the beam matrix or in a special misalignment table. If the beam matrix is used, the results of the misalignment of all magnets in all coordinates will be lumped together, to give an aggregate result. The misalignment parameters may then be fit via constraints on the beam matrix. The misalignment table consists of altered facsimiles of the beam matrix, reproduced a number of times. If the misalignment table is used, then the results of misaligning each magnet in each coordinate can be shown individually.

TURTLE

TURTLE is a Monte Carlo program used to simulate beam line performance. It can produce histograms and scatter plots showing beam profiles and any distribution or correlation of any physical quantity. It also includes second-order and many higher-order terms and aperture constraints.

The input deck is the same as that used for TRANSPORT. Three changes are required to make it into a deck for running TURTLE. First, the computer must be instructed to run TURTLE instead of TRANSPORT. Second, the TRANSPORT indicator card indicating whether the problem is new or a continuation of an old problem is changed to the number of rays to be run through the system. Third, the histogram requests must be inserted.

The transformations through quadrupoles, sextupoles, and solenoids are done individually for each momentum. The geometric effects on trajectories are limited to second order. For a general bending magnet, a second-order transformation in both chromatic and geometric effects is used. The pole face rotations with their accompanying fringe fields, and the body of the bending magnet are all included in a single second-order transformation.

If the field gradient of a magnet is zero, then the trajectory of a particle through a magnet will be the arc of a circle. A rectangular magnet with a sample circular trajectory is shown in figure 6. If α is the bend angle of a sector bending magnet, then the output coordinates expressed in terms of the input coordinates are

$$\sin\theta_1 = \sin\theta_o\cos\alpha - \frac{\sin^2\theta_o\sin\alpha}{1+\cos\theta_o} - x_o\frac{\sin\alpha}{\rho} + \frac{\delta}{1+\delta}\sin\alpha$$

$$x_1 = x_o\left[\cos\alpha + \frac{\sin\alpha(\sin(\theta_o+\alpha)+\sin\theta_1)}{\cos(\theta_o+\alpha)+\cos\theta_1}\right]$$

$$+ L\frac{\sin\alpha}{\alpha}\frac{(\sin\theta_o+\sin\theta_1)}{(\cos(\theta_o+\alpha)+\cos\theta_1)}\left[1+\frac{\sin\alpha}{1+\cos\alpha}\ \frac{\sin\theta_1-\sin\theta_o}{\cos\theta_1+\cos\theta_o}\right]$$

$$y_1 = y_o + y_o'\frac{\cos\theta_o}{\cos\frac{1}{2}(\theta_o+\alpha+\theta_1)}\ \frac{\frac{1}{2}(\alpha+\theta_o-\theta_1)}{\sin\frac{1}{2}(\alpha+\theta_o-\theta_1)}\left[x_o\sin\alpha + L\frac{\sin\alpha}{\alpha}\right]$$

$$y_1' = y_o'\frac{\cos\theta_o}{\cos\theta_1} \tag{13}$$

The quantity L is the length of the bend magnet, ρ the radius of curvature of the individual trajectory, and θ_o the inverse tangent of x_o'. For a rectangular zero-gradient bending magnet, the effect of the pole face rotations is included via a second-order transfer matrix. Also for a rectangular bending magnet, the proper sagitta offset and corresponding aperture limitations can be calculated automatically.

The effect of non-linearities in quadrupole fields may also be calculated automatically. The magnitude of the error field is represented by a multipole expansion

$$B_x = \sum_{n=1}^{\infty} B_{no} \left(\frac{r}{a}\right)^{n-1} \sin\left[(n-1)\theta - \alpha_n\right] \qquad (14)$$

$$B_y = \sum_{n=1}^{\infty} B_{no} \left(\frac{r}{a}\right)^{n-1} \sin\left[(n-1)\theta - \alpha_n\right]$$

The mathematical procedure for calculating the effect of the error field has been described elsewhere.[9] Briefly, the trajectory is transformed to the longitudinal center of the quadrupole by an ordinary transfer matrix. At that point it is perturbed by an integral of the multipole field, where the strengths of the conponents are determined from the magnitude of the ray at the longitudinal midpoint. Then it is transformed to the end of the quadrupole. Multipoles up to the 40-pole may be included.

Replacable subroutines allow a variety of input phase space distributions, scattering distributions, slit shapes, and histogrammable variables. The normal phase space is rectangular in x vs. x' or y vs. y', but circular in x vs. y or x' vs. y'. It is also uniform in momentum. Alternate distributions representing particle production models have been used.

Scattering may be introduced at any point in the beam line. The default distribution is the same as for the input phase space. The subroutine producing the scattering may be replaced by one giving a Gaussian or other desired distribution.

Slits of any size and shape can also be inserted anywhere in the beam line. The program allows a slit in any of the trajectory coordinates. A replacable subroutine allows any other slit that can be formulated mathematically. This feature is useful in defining irregularly shaped apertures of magnets.

Histograms and scatter plots of trajectory coordinates may also be produced. A histogram of the longitudinal position of beam loss is also available. Replacable subroutines allow calculation of any quantity which is related to the trajectory coordinates for inclusion in histograms or scatter plots. Examples might be kinematic variables associated with a particular reaction, the angle of a cone of Cerenkov light, or a reconstruction of an initial trajectory coordinate from chamber readings. The scatter plot may contain any two variables, which need not occur at the same location in the beam line.

Histograms and scatter plots may be flagged. A trajectory coordinate will not be entered into the histogram unless the ray remains in the beam when the flag is encountered. This feature is especially useful for determining the effective acceptance of a beam line.

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Element	Can Vary
Initial beam ellipse	All dimensions
Pole face rotation	Angle of rotation
Drift space	Length
Bend magnet	Length, field, field gradient
Quadrupole	Length, pole tip field
Initial beam centroid shift	All dimensions
Alignment tolerance	All dimensions
Accelerator	
Arbitrary Matrix	All elements
Initial floor coordinates	All positions and angles
BM sextupole components	Strength
Sextupole	Field
Solenoid	Length, field
Coordinate rotation	Angle of rotation

Table I. Physical and Variable TRANSPORT Elements

<u>Zero'th Order</u>

Total system length

Floor coordinates -- three positions and three angles

<u>First Order</u>

Transfer matrix element -- R_{ij}

Beam size in any coordinate

Beam matrix element -- σ_{ij}

Beam correlation matrix element -- $r_{ij} = \sigma_{ij}/\sqrt{\sigma_{ii}\sigma_{jj}}$

First moments of the beam -- centroid

Phase advance -- Trace of R matrix in one plane

<u>Second Order</u>

Transfer matrix element - T_{ijk}

Second order contributions to beam size in any coordinate

Table II. Fitting Constraints

Figure Captions

Fig. 1: A charged particle beam line, with the reference trajectory shown.

Fig. 2: The local coordinate system for determining the local coordinates of a particle trajectory.

Fig. 3: A two-dimensional beam phase ellipse.

Fig. 4: Field boundaries for a general bending magnet.

Fig. 5: Perfectly aligned and misaligned bending magnets.

Fig. 6: A zero-gradient rectangular bending magnet. The reference trajectory and another sample particle trajectory are shown.

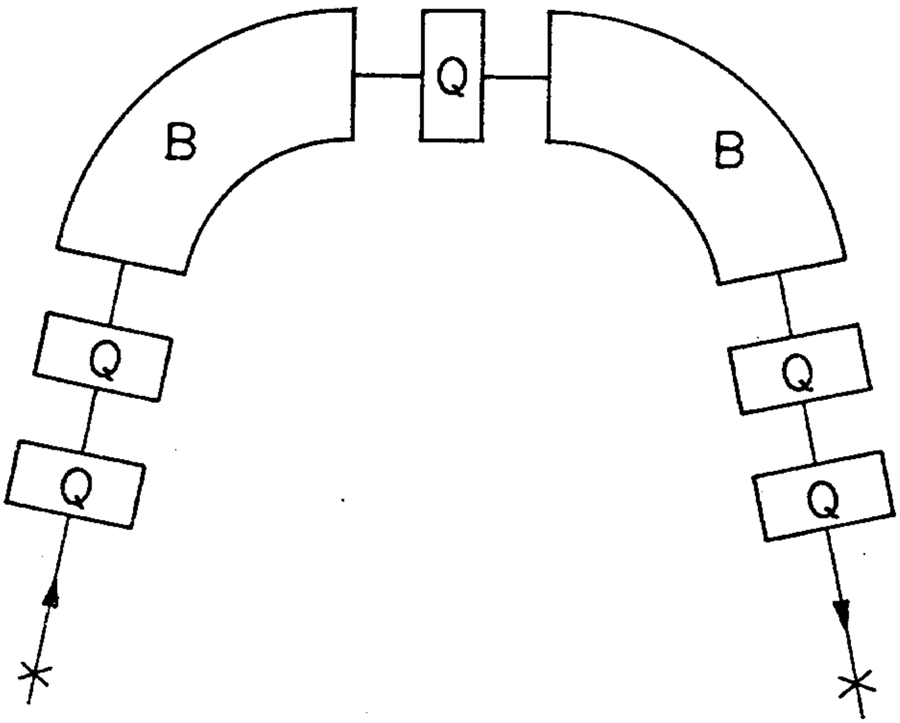


Figure 1.

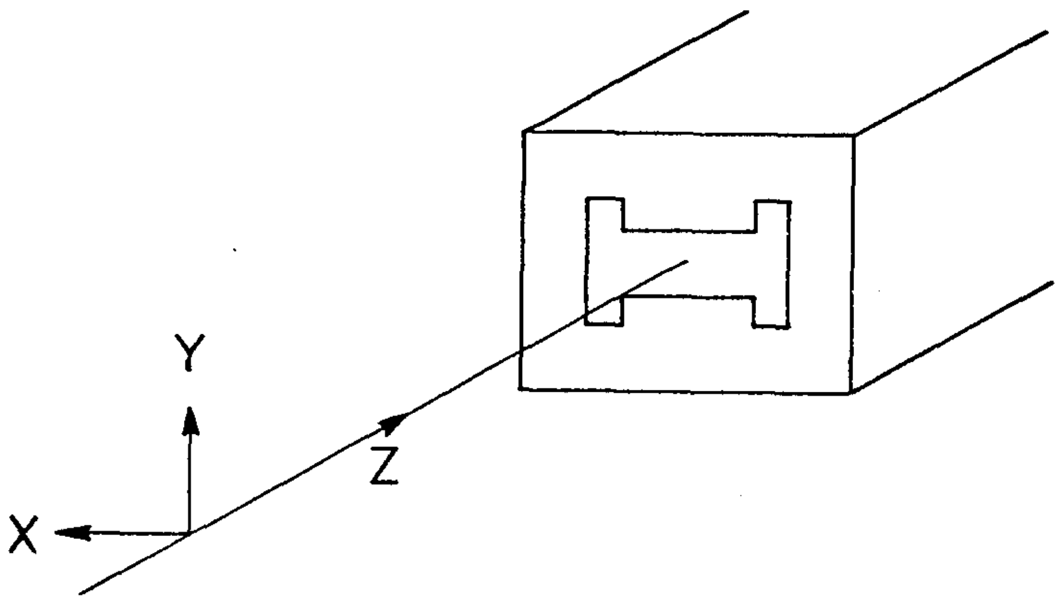
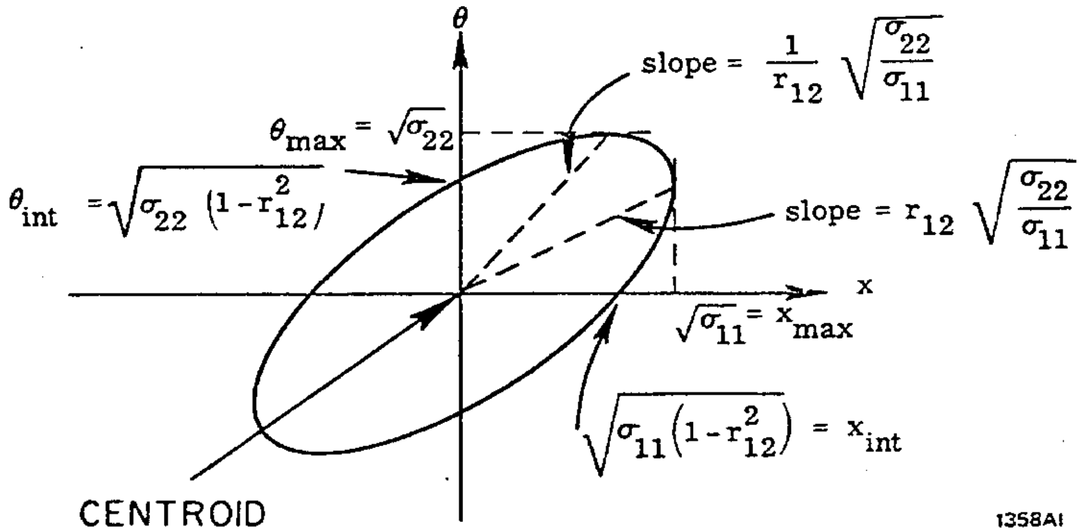


Figure 2.



θ
slope = $\frac{1}{r_{12}}\sqrt{\frac{\sigma_{22}}{\sigma_{11}}}$
$\theta_{max} = \sqrt{\sigma_{22}}$
$\theta_{int} = \sqrt{\sigma_{22}\left(1-r_{12}^2\right)}$
slope = $r_{12}\sqrt{\frac{\sigma_{22}}{\sigma_{11}}}$
x
$\sqrt{\sigma_{11}} = x_{max}$
$\sqrt{\sigma_{11}\left(1-r_{12}^2\right)} = x_{int}$
CENTROID
1358A1

Figure 3.

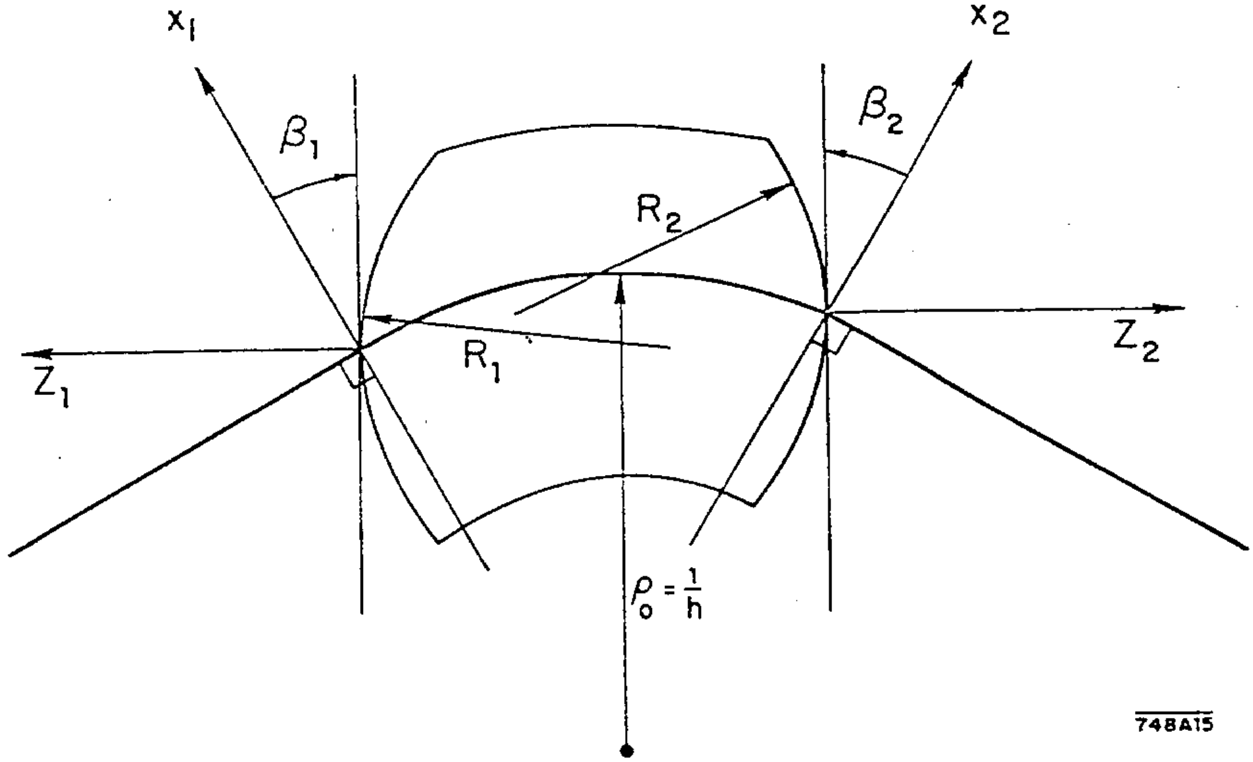


Figure 4.

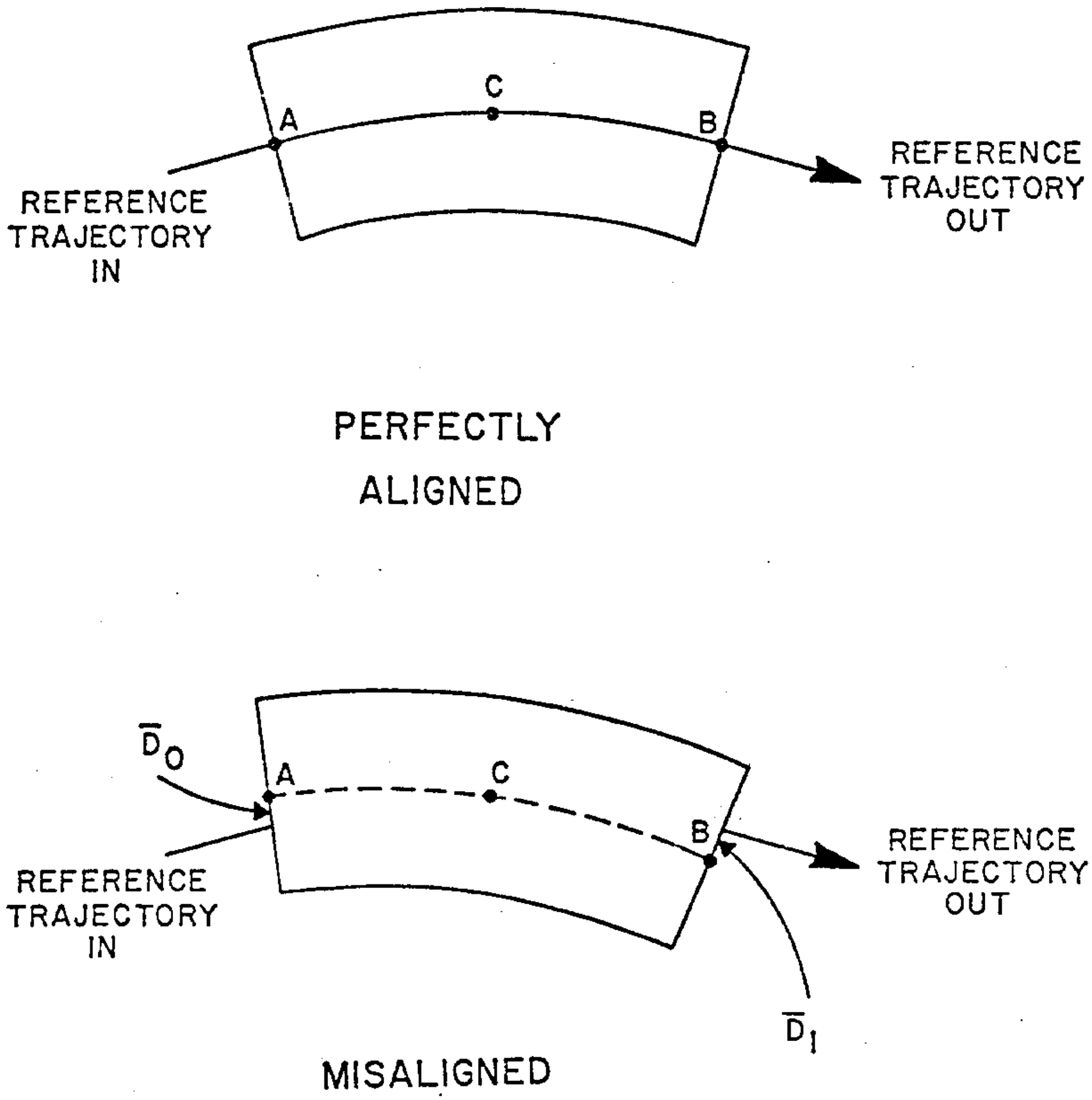

C
A
B
REFERENCE TRAJECTORY IN
REFERENCE TRAJECTORY OUT
PERFECTLY ALIGNED
$\bar{D}_O$
A
C
B
REFERENCE TRAJECTORY IN
REFERENCE TRAJECTORY OUT
$\bar{D}_I$
MISALIGNED

Figure 5.

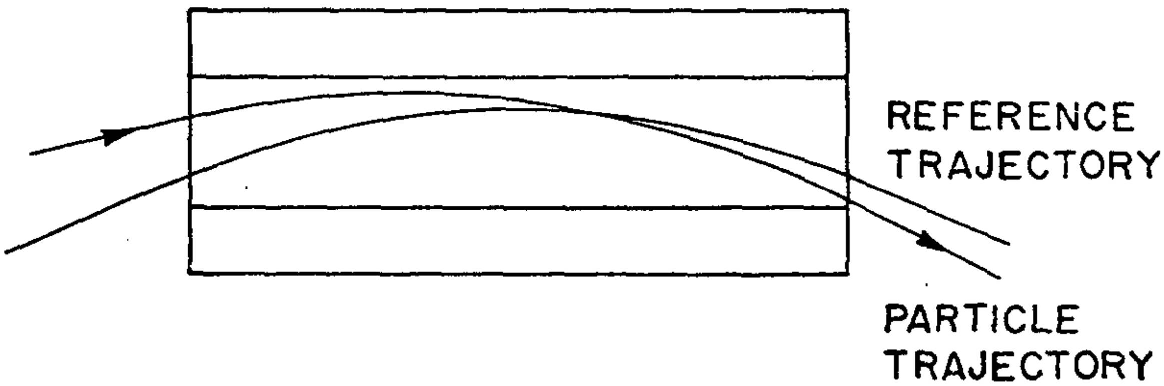


Figure 6.